

DOI: https://doi.org/10.48009/4_iis_2026_130

Marketing Introduces Uncertainty into Programmatic Advertising: An analysis with Shannon Entropy

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Abstract

This paper examines the use of Information Theory, specifically Shannon entropy, in programmatic media buying. Probabilistically guided systems introduce uncertainty to the steps of their processes and into outcomes. Shannon entropy can be used to measure uncertainty in processes and has been used in various fields to find valuable information in noisy environments; however, it has not seen widespread use in marketing automation systems.

An R language simulation was prepared to model the operation of a programmatic media buying operation and the four primary operations taking place in this type of marketplace. The simulation is used to investigate the relationship between conversion rate and uncertainty in this study. Five different scenarios were used to watch the relationship across various campaign situations.

The finding is that greater improvements in conversion also show higher levels of entropy. Although it seems counterintuitive, in these environments, as uncertainty rises, the conversion rate tends to rise. The impact of marketing activities is to move the initial system state from one of high certainty of no conversion to a state with less certainty about conversion failure, i.e., more uncertainty. This rise in uncertainty reflects the disruption marketing causes to the system's initial state, which has a high certainty of failure.

The findings apply to programmatic campaigns executed when conversions are rare relative to total impressions. The conclusion is that using Shannon Entropy provides an additional insight into understanding campaign performance, complementing other performance measures. It also opens additional avenues of investigation on how other aspects of information theory might provide new insights into campaign performance, both retrospectively and in the real-time world of programmatic media buying.

Keywords: programmatic advertising, Shannon entropy, Information Theory, marketing campaign dimensions, logistic equation

Introduction

Programmatic Advertising

Programmatic media buying is an automated way to purchase ad placements, guided by data, to deliver the right ad to the right person at the right time, all in real time. The actual transactions for purchasing an ad slot and placing an ad impression are fully automated (McGuigan, 2019). Configuring the marketing campaign parameters is typically manual, and involves inputting budgets, defining constraints or audience segments, and setting bid strategy (Asadpour et al., 2019).

There are two different environments where programmatic media buying takes place: the open web and social media. Even so, ad delivery follows a common process: a visitor interaction generates an ad

opportunity, the platform evaluates eligible campaigns using visitor and contextual data, an automated auction selects the winning ad based on bid and performance signals, and the selected creative is rendered on the page visited (ARTF, n.d.).

The following steps represent the mechanics of the programmatic media buying process:

- A person opens a social media app or visits a website
- The programmatic media service identifies an available ad opportunity, such as a slot on a web page or a feed position in social media
- The service evaluates user, context, and eligibility using internal data (accessing external data would be too slow)
- The service retrieves eligible advertiser campaigns
- The service requests those campaigns to bid in an automated auction
- The winning campaign selects an ad creative that is formatted for the visitor's environment
- The ad is rendered in the social media feed or on the requested web page
- Client-side code enables performance monitoring

As an example, a person browsing the web requests a web page with an ad slot from a premium travel website. Before the web server sends the page to the person's browser, it notifies a server-side platform (SSP), a service that assists the travel website in selling its ad space. The SSP creates a bid request with visitor data such as user identifiers, device ID, and context data on the nature of the website. The SSP fans out the request to subscribing demand-side platforms (DSPs) or to ad exchanges that forward it to their affiliated DSPs. The DSP is a service that assists advertisers in bidding on and placing their ads in appropriate ad slot offerings.

The DSP retrieves suitable advertiser campaigns and submits bids to the SSP-initiated auction. Continuing with the example, a luxury car advertiser campaign wins the bid. The ad creative is dynamically assembled, and the ad is rendered as the requested page is delivered to the browser. The person has been waiting for around 100 ms (Voisetechn, 2025).

Shannon Entropy

One of the most influential equations of the 20th century was $H = -\sum p \log(p)$. Shannon's Information Theory introduced the concepts of quantifiable information and uncertainty, in addition to describing a mathematical law (Stone, 2023). This theory has had wide-ranging applicability across fields such as statistics, genetics, biology, economics, signals, and linguistics. Of course, it has been applied in the high-tech industry that developed rapidly in the 20th century.

A key element in Information Theory and an essential starting point is uncertainty. Originally, Shannon called the concept uncertainty, but was later convinced to call it entropy, now known as Shannon entropy (Vasiloudis, 2018). An important question to ask in probabilistically guided systems is the level of uncertainty. Programmatic media buying is a probabilistically guided system, yet the application of information theory to its operation and performance analysis is not widespread (McGuigan, 2019).

Its usefulness for programmatic media buying is apparent. In general, marketing automation systems deal with random variables, such as user behavior and purchase intent (Sun & Morwitz, 2010). Programmatic media buying operates in a noisy environment with cookies, pixels, contextual data, and bid responses (Haglund & Bjorklund, 2022). In this system, decisions are repeatedly made in conditions of uncertainty. For example, should a bid be made by this campaign for a presented ad slot, and, if so, for how much? There are channel capacity constraints on available inventory types at any given moment, brand safety issues, and response time thresholds (Shehu et al., 2020).

Uncertainty can be used in performance monitoring. Measured uncertainty can be compared with other performance measures to discover relationships and acquire insights into the nature of programmatic advertising dynamics. This paper investigates the relationship between conversion and uncertainty. In Information Theory, a bit is sufficient information to know which of two equally probable alternatives, 0 or 1, has resulted (Stone, 2023). Two such results occurring in sequence can be represented by two bits: a 0 or 1 for the first result, and a 0 or 1 for the second. If a third result is sequenced with the first two, then 3 bits are required. The event space for 1 result is 2; for a sequence of two results, it is 4. Adding another sequence to the already presented sequences multiplies the event space by the number of possible alternatives in the latest result. In the case of a third result for two equally probable alternatives, the event space is $4 * 2 = 8$.

If each result has the same number of alternatives, n , then given an event space of M , the number of bits needed to present the results is $\log_2(M)$. In the case of 2 alternatives for each result and an event space of 8, the number of bits required to report the results is $\log_2(8)$ or 3. There is more knowledge stored in 3 bits because they report 1 out of 8 possible outcomes than in 2 bits because they report 1 out of 4 outcomes. If p is the probability of an event happening, p would equal 50% in the case of two equal-probable outcomes for an event. The inverse of p , expressed as $1/p$, represents the expected number of trials required to see the event occur (Bogomolny, n.d.).

There is more surprise in signals with a low probability of occurrence than high. For example, someone says something about Nike shoes that no one else is saying. If 999 out of 1000 messages say Nike is great and 1 out of 1000 say they wear Nike when skimboarding, this might be worth looking into. If everyone is saying the same thing about Nike, i.e., it has a high probability of occurrence, what they're saying is probably not surprising.

Shannon entropy could also look at patterns in signals, for example, a visitor's search history starting to focus on a single topic related to the sale of a product, rather than a wandering search history. Tracking and topic modeling the various aspects of context will be increasingly important (Chen, et al, 2018; Reisenbichler & Reutterer, 2019).

Contribution

The use of entropy quantifies the level of uncertainty in the outcomes in programmatic advertising. Traditional measures such as conversion rate tells how often users convert; click-through rate tells how often users click; and reach tells how many users are exposed (Gharibshah and Zhu, 2021). The principal contribution of this paper is introducing Information Theory as an additional analytical framework for understanding programmatic advertising. Shannon entropy complements traditional measures by quantifying the uncertainty associated with conversion outcomes, providing an additional perspective from which campaign dynamics can be analyzed.

Programmatic Advertising: an Information-Processing System

Programmatic advertising is a probabilistic decision system with the effects from marketing activities changing click and conversion probabilities. This system will allocate more impressions to combinations of audience segment, usp, inventory and device that have observably higher click and conversion probabilities. Information Theory studies the uncertainty in probabilistic systems. It follows that Information Theory should find useful applications in programmatic advertising.

The relationship between probability and Shannon entropy follows directly from Information Theory. The contribution of this research is demonstrating the practical implications of that relationship within a

programmatic advertising environment and showing how entropy may be interpreted as an operational metric for understanding campaign dynamics when conversion probabilities are low.

Methodology

Research Question

The hypothesis is that there is a variation in the correlation between conversion rate and outcome uncertainty that follow a pattern. This variation provides useful insights into programmatic advertising. An R simulation is used to test the relationship between them. This is the first in a series of research projects into the operational relationships in a programmatic media buying system.

Overview

A Monte Carlo simulation of a programmatic advertising environment is used to illustrate how Information Theory provides another analytical lens for studying programmatic advertising. A series of base logits, ranging from -2 to -6 are used to represent markets with different baseline conversion probabilities in the simulation. Each simulated impression has four marketing dimensions: audience segment, USP, inventory type, and device. These dimensions have randomly assigned effect sizes within predefined ranges and predefined interaction effects are assigned for selected dimension combinations. Sums of these effects with the base logit produce an impression-level conversion logit.

Each logit is transformed into a conversion probability using the logistic function, and binary entropy is calculated from that probability as a measure of uncertainty. The program then computes Pearson correlations between entropy and conversion probability, and between entropy and the total marketing logit effect, including confidence intervals, and significance tests. Sensitivity analysis is performed by repeating the experiment over a range of base logits (-2 to -6). Aggregate results are summarized using tables and figures illustrating how mean entropy varies with both baseline and realized conversion probabilities.

A simulation is necessary because it provides the ability to change market assumptions to see the impact that they have on certain relationships. It allows the base logit to be deliberately varied across five different conditions. This cannot be done with observed data. This study intentionally isolates the entropy-conversion relationship under the pristine conditions identified by the reviewer. The simulation is based on independent dimension sampling, known effect sizes, and the absence of auction-level confounds such as bid shading, budget pacing, and creative fatigue. These controlled conditions were deliberately chosen to establish the baseline mathematical relationship between uncertainty and conversion probability before introducing the additional sources of variability present in operational programmatic advertising markets. This study establishes the baseline relationship under pristine conditions. Future work will introduce additional marketplace dynamics to evaluate whether the relationship remains robust in increasingly realistic programmatic advertising environments.

Campaign Dimensions

An R language program provides the simulation of a programmatic media buying system, and four random variables represent that market system: 1) audience; 2) USP; 3) device; and 4) inventory. These represent the four rights of programmatic: the right person, the right message, the right place, and the right time. These 4 R's also map onto the actual decision variables in a DSP: 1) audience targeting; 2) creative/USP selection; 3) device targeting; and 4) inventory/placement. These become categorical random variables assigned a value at each trial.

The application generates 50,000 simulation records by randomly selecting values for each random variable and storing them in a dataframe. In a marketing campaign, the values in the four dimensions would not be uniformly available or have uniform utility. In this simulation, weights are assigned to reflect availability and utility. These weights reflect how often a value will be selected when forming the 50,000 simulation records.

Logit

In this programmatic media buying simulation, changes in conversion probability are achieved through marketing activities such as selecting appropriate audience segments, choosing appropriate unique selling propositions, selecting inventory contexts, and considering interactions, such as the right USP for the segment or the right creative for the device. In addition, a conversion is considered a binary outcome (yes/no). The logit (log-odds) transformation maps probabilities in the 0 to 1 range to the real number range, supporting additive modeling of effects and the application of linear statistical techniques. There is a linear relationship between input features and the log-odds of a logit (JMP, n.d.). The logistic function is used to transform logit values back to probabilities, preserving outcome probabilities. This makes it a natural bridge between marketing effects and conversion outcomes (Song, 2023).

It is important to note that while logits are used for convenient arithmetic operations, probabilities are still essential for calculating the conversion outcome (Frick et al, 2023). In the simulation, conversion probabilities vary along the four dimensions of this simulation. This makes it a natural basis for measuring Shannon entropy and mutual information: quantifying how much each dimension and the interactions between dimensions change uncertainty.

Dimension Effects

An important aspect of the methodology is to establish a system for applying the effects of different marketing actions on conversion. Each of the four random variables that define a programmatic media buying operation influences conversion, as do certain interactions. Audience segmentation stands in contrast to mass marketing, and developing a USP stands in contrast to a commodity approach. Carefully considering ad inventory context and analyzing device data stand in contrast to blind buying. The intent of these marketing activities is to increase conversion, and, for each of these dimensions, the simulation system must provide for interactions among them (Frick et al, 2022).

The core of this system of effects in the simulation is a starting logit and effect logits assigned to each level of the random variables, as well as interacting effects for certain combinations of their levels (Fox & Andersen, 2006; Alaimo & Kallinikos, 2018). All these will be stored in a dataframe for analysis. This study aims to compare Shannon entropy and conversion and to examine the impact of marketing activity on the conversion start-logit.

A list of abstract labels is set up for each dimension. For example, for right person there are four segments, SEG1 through SEG4. Applying to a specific market, the EV automobile market, the abstract label SEG1 might be replaced with social currency (what others think) buyers, SEG2 with environmental buyers, SEG3 with economic buyers, and SEG4 with social externalities (incentives like tax breaks) buyers. Different numbers of labels can be assigned to each dimension, but in this simulation, the essential element is the ability to randomly select different labels from a dimension and assign effects on conversion to those labels, as a general representation of the system operations.

To examine the dimensional effects, averages of conversion probabilities across case studies for those four dimensions are used. In the simulation, the probabilities are randomly selected from case study ranges, converted to logits and assigned to one of the abstract labels at each of the 50,000 trials. For interaction effects, not all pairings have equal impact.

Four interaction effects are considered in this study: 1.) audience with USP; 2.) audience with device; 3.) device with inventory; and 4.) USP with inventory. These effect interactions reflect the right message reaching the right person, audience device preferences, inventory available on a device, and a creative message with inventory context. Because the four dimensions are independently sampled, the simulation avoids structural multicollinearity among audience, USP, inventory, and device assignment. As Shannon entropy is calculated solely from the resulting conversion probability from the marketing activities, it is assumed that the interaction effects do not introduce multicollinearity into the entropy calculation.

The effect logit assigned to each arbitrary label is based on empirical data. Many case studies on conversion effects report ranges, while others report point increases resulting from the marketing activity. The effects of audience segmentation compared with mass marketing is to improve conversions from 20 to 50% (Lumino, 2025; OnRamp, 2025). The effects of a unique selling proposition on conversions range from 20% to 60% (McKinsey, n.d.). Goyal (2024) reviewed 9 case studies, and the case study that focused on USP saw conversion rate improve 31%.

The effects of inventory vary by type. For CTV, conversion rates increase up to 22.3% when CTV is added to the marketing mix (Paramount Ads Manager, 2026). Display ads increase conversion by up to 52% (Location3, 2022). Programmatic ads can increase conversion by 10 to 30% (Nash, 2026a). UGC increases conversion up to 40% (Nash, 2026b). For each of the 50,000 trials in the simulation, these marketing effects will be randomly selected from the just discussed ranges (Goltz & Northey, 1998).

These effects represent current industry estimates. This simulation is using these ranges as plausible parameter values to create a synthetic marketplace, rather than for estimating marketing effects from data. Industry reported values can be well suited for this purpose because industry has access to enormous datasets and has an economic incentive to maintain those datasets.

The results of each trial are stored in an R dataframe. A correlation between uncertainty and conversion is calculated using data from that dataframe. Five different starting scenarios are used in the simulation, including a high conversion probability of 4.7%, a medium conversion probability of 1.7%, and a low conversion probability of 0.67% (eMarketer, 2024).

Results and Discussion

This R simulation shows that in programmatic advertising, with a low probability of conversion, there is a strong positive correlation between entropy and conversion probability. In other words, as uncertainty increases, conversion rates increase. The result contradicted initial expectations.

Table 1: Simulation Results

Start Probability	Start Logit	Correlation: Uncertainty and Conversion	Precision
11.92%	-2	.980	95% CI [0.9801, 0.9808] significant at alpha = 0.05
4.7%	-3	.995	95% CI [0.9951, 0.9953] significant at alpha = 0.05
1.7%	-4	.998	95% CI [0.9978, 0.9978] significant at alpha = 0.05
.67%	-5	.999	95% CI [0.9990, 0.9991] significant at alpha = 0.05

.25%	-6	.999	95% CI [0.9993, 0.9993] significant at alpha = 0.05
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Notes: Table 1 shows the starting probability of conversion, also expressed as a logit, and the resulting Pearson correlation between uncertainty and conversion from marketing activities simulated with 50,000 trials

The explanation for these results is that marketing starts with a low probability of conversion. In this study, the lower conversion probabilities in the five scenarios were 0.67%, 1.7%, and 4.7%. These have low entropy or uncertainty; they indicate a high level of certainty that a sale won't be made. The marketing campaign raises that outcome uncertainty, meaning there is less certainty of failure, because the marketing effects raise the probability of sales. Entropy increasing in the low-conversion environment of programmatic advertising may represent a disruption of the initial state facing the advertiser, indicating a campaign has increased the possibility of conversion. Another viewpoint is visualized in Figure 1.

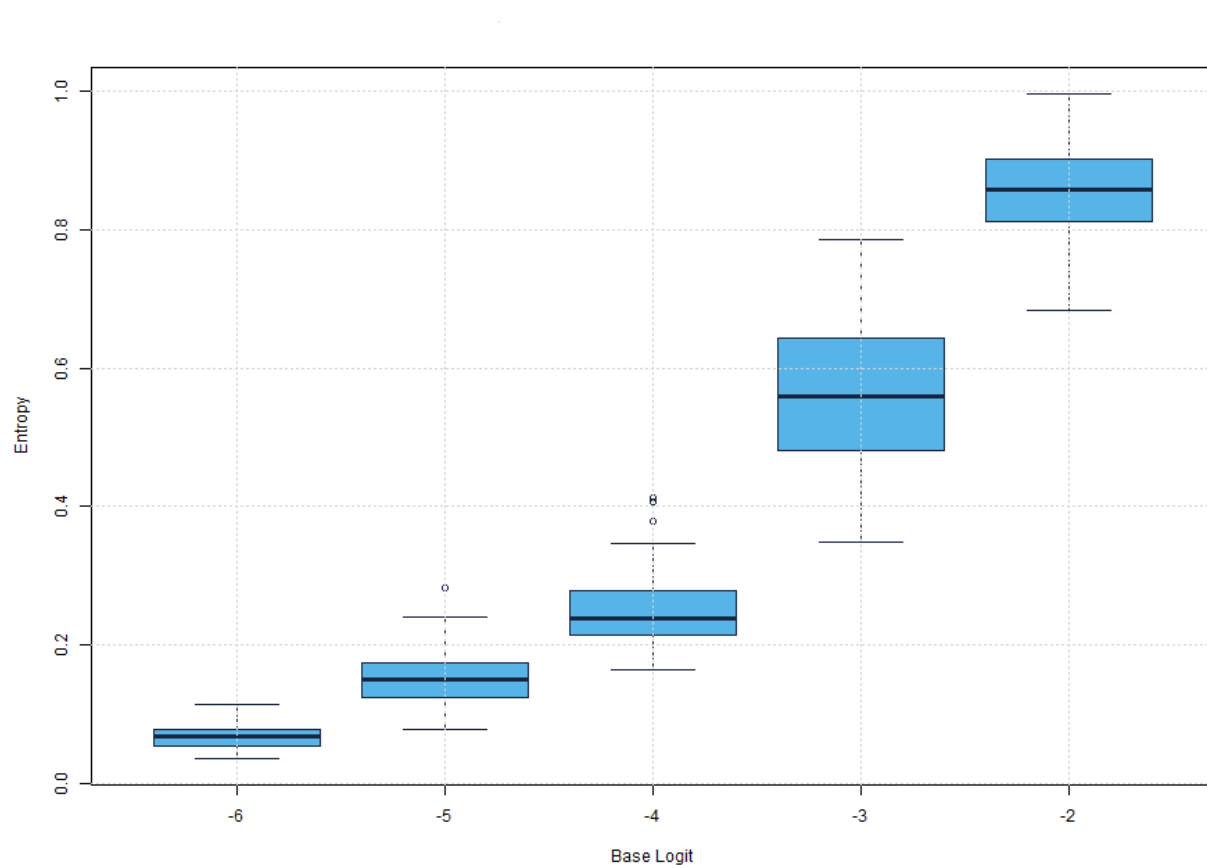


Figure 1: Box Plot of Entropy Distribution by Base Logit

As the base logit increases from -6 to -2, the distribution of entropy shifts upward nonlinearly. The boxplot also shows that entropy increases monotonically as the base logit rises from -6 to -2. This is expected as the base logit increases from -6 to -2 because higher base logits produce higher conversion probabilities, which in turn move outcomes away from near-certain non-conversion and toward greater uncertainty.

The dispersion of entropy values generally widens over most of the range in the box plot as the base logit increases, indicating that marketing effects and interaction effects create a wider range of uncertainty when the baseline conversion environment is less sparse. At the lowest base logit (-6), entropy values are

compressed because most impressions remain very low-probability events, so outcomes are consistently close to non-conversion certainty. On the other hand, at the high end, the move from -3 to -2 produces a large increase in overall uncertainty, but the incremental spread from marketing effects is relatively compressed because entropy is limited above by 1 and in this region, entering the high-uncertainty region, it starts recompressing.

For robustness, the base logit was varied from -6 to -2 and mean entropy increased monotonically with starting conversion probability, which is consistent with the theoretical binary entropy function. This supports the use of entropy as an uncertainty measure across low- and moderate-probability PMB scenarios. The Figure 2 shows this relationship.

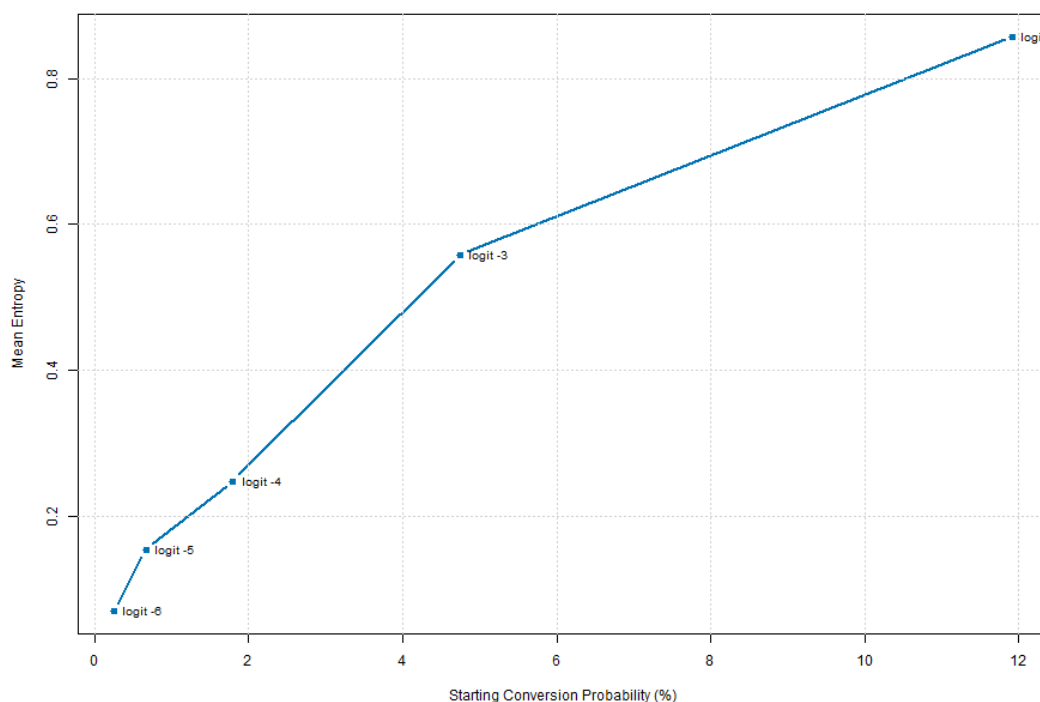


Figure 2: Scatterplot of Starting Conversion Probability and Mean Entropy

The range starts on the low-probability side of the entropy curve. At very low conversion probabilities, outcomes are highly predictable. Because almost no impressions result in a conversion, entropy is low. As conversion probability increases, conversion/non-conversion outcomes become less certain, so entropy increases. The scatterplot is concave: it increases more sharply at low probabilities, and starts flattening as probabilities increase.

Although Shannon entropy for a binary outcome is a deterministic function of conversion probability, the way realistic marketing and interaction effects distribute conversion probabilities across a simulated campaign population is not derivable from the entropy function alone. Figure 1 illustrates this: the dispersion of entropy values widens as the base logit increases, indicating that marketing and interaction effects produce a wider range of outcome uncertainty when the baseline conversion environment is less sparse. This is an empirical regularity that depends on how the simulated dimension and interaction effects combine with the base rate, not on the shape of the entropy function itself.

Figure 2 shows a second, related pattern. While the concavity of the entropy-probability relationship is a known mathematical property, the scatterplot shows that realistic combinations of marketing and interaction effects place most simulated campaigns in the steep, low-probability region of that curve, where small increases in conversion probability correspond to comparatively larger increases in uncertainty. This suggests that marketers operating in typically sparse conversion environments should expect uncertainty to rise disproportionately fast as targeting and creative improvements raise conversion probability from a low base.

Marketing activities act as a disruptor of the dominant failure state, breaking apart the resistance to conversion and introducing the prospect of success. In programmatic advertising, successful marketing activities that increase conversion result in increased outcome uncertainty, because their goal is to upset the initial state, which is almost certain failure.

The outcome holds within the operating range of programmatic advertising, where conversion probabilities are well below 50%. The relationship held over several different random instantiations of the system. After this confirmation, the author used the `set.seed()` function in R so the results as reported in this article are exactly reproducible.

The relationship between marketing conversion probability and Shannon entropy was not apparent to the author before the simulation. Before the simulation, the expectation was that as uncertainty declined, conversion would increase. This simulation clarified that entropy measures the uncertainty of outcome.

It also provides other useful insights. First, an effective programmatic campaign is a process to increase outcome uncertainty when the initial state has a low probability of conversion. Second, it's now apparent that entropy and marketing conversion are connected through the mathematics of Information Theory and that not only is Shannon entropy a meaningful analytic metric for understanding campaign performance, but other metrics like channel capacity and surprisal might likewise be applied. Surprisal may be especially useful in programmatic advertising because unexpected or unusual conversions carry more information than expected ones.

Limitation and Future Work

This study was based on a simulation. A series of studies based on real DSP campaign data would further support the application of Information Theory to programmatic advertising. The simulation program itself can be changed to read DSP API data. This would require permission of the advertiser and DSP administrator in order to access the API. Likewise, it can be adapted to any individual product and marketing campaign by measuring performance and calculating effect logits from those performance results. The list of abstract labels can be replaced by those specific to a particular product. Not only can the abstract names be replaced, but also the number of levels for each random variable and the effect each level has on conversion (Li, et al, 2018).

The interactions can likewise be customized for a specific product and campaign. Furthermore, entropy can be monitored in real-time during campaigns, instead of retrospectively, to investigate other research initiatives about programmatic media buying. One such initiative is applying cross-entropy to measure the gap between historic probability distributions derived from a Customer Relationship Management system or a Customer Data Platform, and current outcomes in the real-time bid processing environment. Another concerns the point of diminishing entropy returns for a product or campaign.

Additional sources of variability such as bid shading, auction floor dynamics, budget pacing, creative fatigue, audience overlap, and inventory constraints will occur in actual programmatic advertising environments. Considerable noise can be introduced by these factors that reduce the strength of the correlations reported in this study. This simulation should be considered as a proof-of-concept application of Information Theory to programmatic advertising, and as a result, the findings should be interpreted as establishing a theoretical baseline rather than as an exact prediction of real-world campaign behavior.

Conclusion

The primary conclusion from this simulation is that effective marketing activities introduce entropy into an event with an initial state having a near-certainty of failure. Although counterintuitive, the simulation revealed a structural property of entropy in a programmatic advertising environment, grounded in the mathematical relationship between uncertainty and conversion. An insight from this is the role of marketing as an agent of disruption, unsettling the market by systematically adding uncertainty to the near certainty of failure. It does this through applying marketing activities categorized as the 4 R's in this simulation, modeled by four random variables.

The four random variables in the simulation grounded the model in standard programmatic advertising practice while also showing that programmatic advertising is an information system subject to Shannon entropy. The findings suggest that Information Theory is applicable to programmatic advertising as a framework to analyze relationships and monitor campaign operations. Shannon's entropy, channel capacity, and surprisal may be useful as analytical tools to understand how a programmatic media buying campaign is performing and to make adjustments during the campaign to optimize results.

The application of Shannon's theory was foundational in the advancement of radar operations by enabling the detection of weak signals in noisy environments. Programmatic advertisers face a similar challenge in detecting conversion signals in the noisy digital data available to them. While these information theory principles have not been widely considered in the marketing field, they have potential in other areas of marketing analytics.

Acknowledgements

Today, it is difficult not to use AI in conducting research and writing the corresponding articles. During the past several years, the large language models have been integrated into search engines, such as Gemini into Google Search. When searching for materials, often an AI overview is provided, which the author reads and evaluates for relevance and accuracy. These overviews provide a summary of the question domain, along with references that support that summary. This AI-enhanced search has helped clarify issues or answer questions much faster than the author would have taken in the past, before the integration of AI.

AI was also used as a source of R syntax for functions and programming-related questions regarding the simulation. In addition, it provided assistance with the literature review. Claude and ChatGPT served as sounding boards for the author's proposed ideas. In this role, they helped the author both validate his interpretations of simulation results and refine his assertion that Information Theory concepts can be applied to programmatic advertising. Nevertheless, all conclusions and interpretations remain the author's own, and the organization, choice of materials, and methods of this paper originated with the author. The AI systems used were Gemini 2.5, ChatGPT 5.5, and Claude Sonnet 4.6.

Data Availability

The R program used to create this simulation is available on GitHub, <https://github.com/gmrwvu/PMB/tree/main>.

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